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Resolving the AVOAz Symmetry Axis Ambiguity

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SUMMARY

Remotely detecting information about fractures and the stress field is an important objective in the development of unconventional and tight hydrocarbon reservoirs. Fractures and stress cause the earth to become anisotropic which is seismically observable. By observing the P-wave seismic amplitude variation with offset and azimuth (AVOAz) it is possible to infer the presence of fractures and their orientation. Unfortunately, the estimate of the fracture orientation is non-unique with two solutions 90 degrees apart. This issue is well known in the case of the near-offset AVOAz inversion, but is also true for the far-offset approximation. In the case of the far-offset approximation, the azimuth ambiguity also leads to biases in the remaining parameter estimates. This paper explores using geologic and rock physics constraints to resolve this issue. A priori information about the horizontal stress field and the form of the anisotropy is used to determine the symmetry axis orientation for both the near-offset and far-offset AVOAz inverse problems.

Introduction

A method to solve the seven-parameter linearized AVOAz inversion in transverse anisotropic media with a horizontal symmetry axis (HTI), and vertical fractures in an isotropic (VFI) background medium is demonstrated. The seven parameters include: three background parameters such as density, P-wave and S-wave impedance reflectivity; and four anisotropic parameters including an orientation parameter. The HTI Rüger equation (1998) is a subset of this problem. One of the key elements in solving the seven-parameter inverse problem is determining the azimuth of the symmetry axis in the case of HTI media, or of the fracture normal in the case of VFI media. For brevity both azimuths are referred to in this paper as the symmetry axis azimuth ϕ_{sym}. The inverse problem is nonlinear with a bimodal solution. The nonuniqueness manifests itself as a 90 degree ambiguity in the estimate of ϕ_{sym} biasing the remaining six parameters. Through the introduction of geologic and rock physics constraints the most likely solution may be chosen.

I begin by reviewing the linearized AVOAz expression written in terms of azimuthal Fourier coefficients (FCs) (Downton et al., 2011) and the parameterizations specific to HTI and VFI media. By writing this in block matrix notation, it is possible to decompose the problem into simpler parts for analysis. The solution of the near-offset linearization is next reviewed with the objective of introducing the symmetry axis ambiguity. It is shown that a priori knowledge of the regional stress field may be used to preferentially choose one solution over the other. Having reviewed the near-offset case, the more complex far-offset problem is discussed and shown to exhibit the same ambiguity. In this case constraints based on the rock physics of fractured media are employed to help resolve the ambiguity. Both synthetic and real seismic data examples are shown to illustrate the method.

Linearized seven-parameter AVOAz

Downton and Roure (2015) write the linearized seven-parameter P-wave AVOAz reflectivity, for HTI and VFI media as the truncated Fourier series

$$R(\phi,\theta) = r_0(\theta) + \mathrm{r}_2(\theta)\cos\left(2\left(\phi-\phi_{sym}\right)\right) + r_4(\theta)\cos\left(4\left(\phi-\phi_{sym}\right)\right). \tag{1}$$

The reflectivity varies as a function of incidence angle θ and azimuth ϕ. In equation (1) the magnitudes of the sinusoids of periodicity n = 0, 2 and 4 are

$$r_0(\theta) = A_0 + B_0 \sin^2\theta + C_0 \sin^2\theta \tan^2\theta, \tag{2}$$

$$r_2(\theta) = B_2 \sin^2\theta + C_2 \sin^2\theta \tan^2\theta, \tag{3}$$

$$r_4(\theta) = C_4 \sin^2\theta \tan^2\theta, \tag{4}$$

where the definitions of the parameters A_0, B_0, C_0, B_2, C_2 and C_4 depend on the form of the anisotropy and are described in Downton and Roure (2015). This paper focuses on the B_2, C_2 and C_4 parameters since they control the Amplitude variation with Azimuth (AVAz). In HTI media $B_2 = 0.5B_{ani}$, $C_2 = 0.25\Delta\varepsilon^{(v)}$ and $C_4 = 1/16\Delta\eta.^{(v)}$ The parameter B_{ani} is the anisotropic gradient, $\varepsilon^{(v)}$ is the Thomsen parameter describing the P-wave anisotropy and $\eta^{(v)}$ represents the anellipticity (Rüger, 2002). All the parameters are evaluated at the interface generating the reflectivity with the symbol Δ denoting the difference operator between the lower and upper medium. The phase of the sinusoids is controlled by ϕ_{sym}. In the case of VFI media, the parameters B_2, C_2 and C_4 are parameterized in terms of fracture weakness parameters. Rotationally asymmetric fractures give rise to orthorhombic anisotropy. The medium is described by the vertical, horizontal and normal fracture weakness parameters δ_V, δ_H, and δ_N respectively. The transformation linking these parameters is

$$\begin{bmatrix} B_2 \\ C_2 \\ C_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}g & 0 & -\frac{1}{2}(1-2g)g \\ 0 & 0 & -\frac{1}{2}g(1-g) \\ 0 & \frac{1}{8}g & -\frac{1}{8}g^2 \end{bmatrix} \begin{bmatrix} \Delta\delta_V \\ \Delta\delta_H \\ \Delta\delta_N \end{bmatrix}, \quad (5)$$

where g is the squared S-wave to P-wave velocity ratio of the background media. The case of rotationally symmetric fractures gives rise to HTI anisotropy. In this case, both the vertical and horizontal fracture weaknesses are equal and are replaced by the single parameter, the tangential fracture weakness δ_T.

AVOAz Inversion

In order to solve the linearized AVOAz inverse problem it is easier to write the Fourier series in terms of cosine (u_n) and sine (v_n) functions. Rewriting equation (1) in block matrix notation

$$\begin{bmatrix} \mathbf{u}_0(\theta) \\ \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \\ \mathbf{u}_4(\theta) \\ \mathbf{v}_4(\theta) \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{x} & \mathbf{z} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{x}\cos(2\phi_{sym}) & \mathbf{z}\cos(2\phi_{sym}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{x}\sin(2\phi_{sym}) & \mathbf{z}\sin(2\phi_{sym}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{z}\cos(4\phi_{sym}) \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{z}\sin(4\phi_{sym}) \end{bmatrix} \begin{bmatrix} A_0 \\ B_0 \\ C_0 \\ B_2 \\ C_2 \\ C_4 \end{bmatrix}, \quad (6)$$

shows that the amplitude variation with offset (AVO) and AVAz parts of the problem are decoupled. In equation (6) all the bold faced vectors are functions of incidence angle θ with $\mathbf{x}=\sin^2(\theta)$, and $\mathbf{z}=\sin^2(\theta)\tan^2(\theta)$. Although written as a set of linear equations, equation (6) is actually nonlinear due to the ϕ_{sym} dependence in the linear operator. A brute force method to solve this system of equations is to iterate over all possible values of ϕ_{sym} solving the least squares problem for each possible ϕ_{sym}. The solution corresponding to the ϕ_{sym} with the minimum misfit is the global solution.

However, the solution of equation (6) is bimodal and hence nonunique. This is more obvious if only the equations describing the AVAz are considered, namely

$$\begin{bmatrix} \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \\ \mathbf{u}_4(\theta) \\ \mathbf{v}_4(\theta) \end{bmatrix} = \begin{bmatrix} \mathbf{x}\cos(2\phi_{sym}) & \mathbf{z}\cos(2\phi_{sym}) & \mathbf{0} \\ \mathbf{x}\sin(2\phi_{sym}) & \mathbf{z}\sin(2\phi_{sym}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{z}\cos(4\phi_{sym}) \\ \mathbf{0} & \mathbf{0} & \mathbf{z}\sin(4\phi_{sym}) \end{bmatrix} \begin{bmatrix} B_2 \\ C_2 \\ C_4 \end{bmatrix}. \quad (7)$$

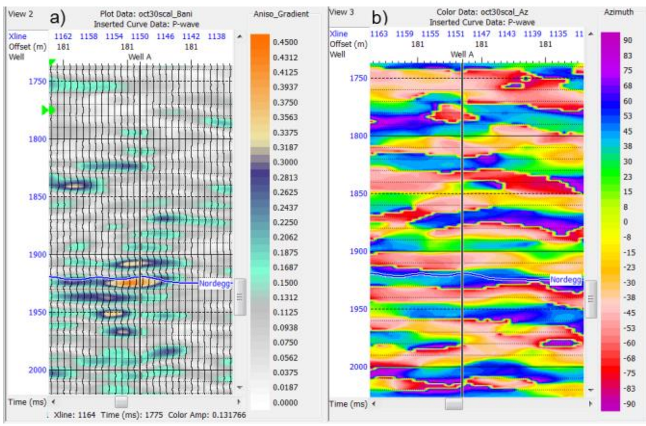

***Figure 1** The (a) anisotropic gradient and (b) symmetry axis azimuth calculated using the positive B_{ani}.*

In the near-offset approximation, the **z** terms are ignored, resulting in

$$\begin{bmatrix} \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \end{bmatrix} = \begin{bmatrix} \mathbf{x}\cos(2\phi_{sym}) \\ \mathbf{x}\sin(2\phi_{sym}) \end{bmatrix} [B_2]. \quad (8)$$

It can be seen by substitution that both $(\hat{\phi}_{\text{sym}}, +0.5\hat{B}_{\text{ani}})$ and $(\hat{\phi}_{\text{sym}} + \pi/2, -0.5\hat{B}_{\text{ani}})$ fit the data equally well. Typically, only one of the solutions is ratained and output. Figures 1a and 1b show the estimated B_{ani} and ϕ_{sym} corresponding to the positive B_{ani} solution for a 3D seismic inline. The azimuth solution oscillates 90 degrees between different layers and hence appears nonphysical. Zoback (2007) notes that the

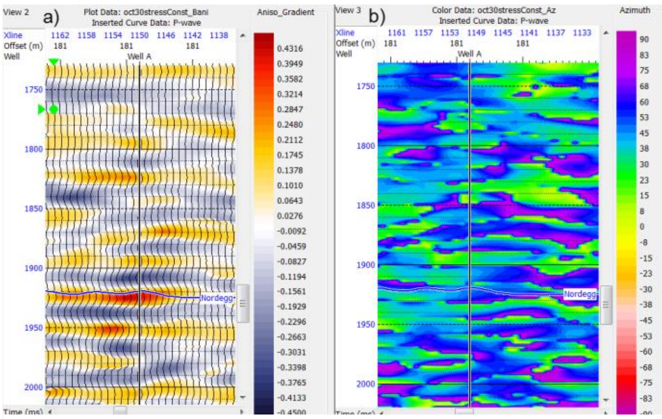

Figure 2 The (a) anisotropic gradient and (b) symmetry axis azimuth calculated using the stress constraint.

horizontal stress field should change slowly in a regional sense. For stress-induced anisotropy the slow direction corresponds to the direction of minimum horizontal stress (i.e. ϕ_{sym}). If this orientation is known from local well control or from the world stress map (Heidbach et al., 2008) then this information may be used to constrain the solution. In this case, the solution is chosen which is most consistent with the minimum horizontal stress direction. The symmetry axis azimuth for this solution is shown in Figure 2b. By definition, it fits with the known geologic information much better. A further consequence is that B_{ani} has both positive and negative values (Figure 2a) which again are more geologically believable.

Similar to the near-offset case, the solution to the far-offset equation (7) has two solutions, $(\hat{\phi}_{sym}, +\hat{B}_2, +\hat{C}_2, \hat{C}_4)$ and $(\hat{\phi}_{sym} + \pi/2, -\hat{B}_2, -\hat{C}_2, \hat{C}_4)$, as can be seen by substitution. Constraints again may be used to reduce the solution space. Downton et al. (2011) assumed the anisotropy is due to vertical rotationally symmetric fractures. Under this assumption, equation (7) becomes

$$\begin{bmatrix} \mathbf{u}_2(\theta) \\ \mathbf{v}_2(\theta) \\ \mathbf{u}_4(\theta) \\ \mathbf{v}_4(\theta) \end{bmatrix} = \frac{g}{2} \begin{bmatrix} \mathbf{x}\cos(2\phi_{sym}) & ((2g-1)\mathbf{x} + (g-1)\mathbf{z})\cos(2\phi_{sym}) \\ \mathbf{x}\sin(2\phi_{sym}) & ((2g-1)\mathbf{x} + (g-1)\mathbf{z})\sin(2\phi_{sym}) \\ \frac{1}{4}\mathbf{z}\cos(4\phi_{sym}) & -\frac{g}{4}\mathbf{z}\cos(4\phi_{sym}) \\ \frac{1}{4}\mathbf{z}\sin(4\phi_{sym}) & -\frac{g}{4}\mathbf{z}\sin(4\phi_{sym}) \end{bmatrix} \begin{bmatrix} \Delta\delta_T \\ \Delta\delta_N \end{bmatrix}. \quad (9)$$

This reduction in the number of free parameters leads to a more stable solution and a global minimum, provided the solution does not exist in the null space. In the more general case of rotationally asymmetric fractures, a similar constraint may be used. In this case rather than forcing $\delta_V = \delta_H$, the solution is chosen in which δ_V and δ_H are closest together. Figure 3 shows the application of this constraint to the inversion of equation (6) on synthetic data. The parameters are transformed using equation (5).

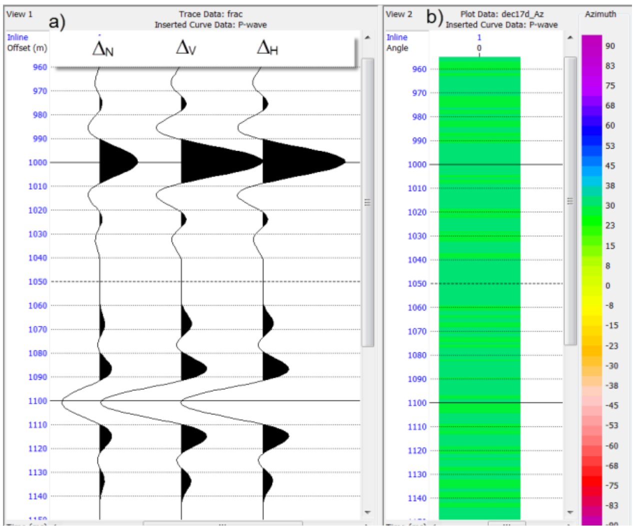

***Figure 3** The (a) normal, vertical and horizontal fracture weakness contrast parameters estimated from the seven-parameter AVOAz inversion based on synthetic data. All show the correct azimuth (b) of 30 degrees. The fracture weakness contrasts are all positive at the top of the fractured zone (1000 ms) and negative at the base (1100 ms) and have the correct polarity.*

Alternatively, other empirical rock physics relationships may be used. The penny shaped crack theory of Hudson (1981) may be used to reduce the number of fracture parameters to a single variable. The sign of B_{ani} then depends on the background g and the fluid content. Another popular approximation is to make the P-wave anisotropy $\varepsilon^{(v)}$ approximately equal to the S-wave anisotropy $\gamma^{(v)}$ (Wang, 2002). These constraints may be used in combination. For example, a rock physics constraint may be used in combination with the stress constraint and some spatial continuity constraint.

The three-parameter AVO inversion is a subset of the seven-parameter AVOAz inversion and thus suffers the same stability issues. In order to get stable three-parameter AVO inversion estimates, seismic data with exceptional signal-to-noise and incidence angles in excess of 45 degrees must be acquired and incorporated. The AVOAz inversion problem has the additional requirement of azimuth sampling finer than 22.5 degrees with at least 8 azimuths. In practice, it is probably more stable to work with reduced parameterizations such as equation (9) and shown in Downton et al. (2011). Alternatively, it might be best to consider this problem as part of a simultaneous azimuthal inversion such as Downton and Roure (2010). In the AVOAz inversion each time sample is treated as an interface and wavelet issues must be dealt with. For example, at the zero crossing of the wavelet the estimate of ϕ_{sym} is unstable. Lastly, it is easier to incorporate greater theoretical complexity in azimuthal inversion, such as allowing the symmetry axis to vary as a function of layer.

Conclusions

Both the near-offset and seven-parameter AVOAz inversions are non-unique and exhibit a 90 degree azimuth ambiguity. This ambiguity biases the remaining six parameter estimates, the size and complexity of which depends on the parameterization. This ambiguity may be reduced by imposing geologic constraints including: the regional stress field, continuity constraints and empirical relations linking the anisotropic parameters. This study showed how to introduce these constraints and also illustrated the solution with both real and synthetic examples.

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