# Least-squares Q migration: the path to improved seismic resolution and amplitude fidelity

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## Summary

Standard prestack depth migration (PSDM), e.g., Kirchhoff/RTM, is by nature unable to fully recover the reflectivity with the desired amplitude fidelity and resolution due to factors such as inhomogeneous subsurface illumination and irregular acquisition geometry. This shortcoming is well recognized by the imaging community and has propelled the re-emergence of least-squares migration (LSM) in recent years. Another factor that degrades the amplitude fidelity and image resolution is the attenuation of seismic waves induced by anelastic absorption and elastic scattering during its propagation inside the earth, quantified by the so-called quality factor (O). Absorption causes frequency-dependent amplitude decay, phase distortion, and resolution reduction. This effect can be compensated through Q prestack depth migration (QPSDM). QPSDM has become an effective solution for seismic imaging in areas where strong absorption anomalies exist in the overburden. However, the excessive noise often accompanying QPSDM poses a big challenge to its application. We describe a least-squares Q migration (LSQM) method that combines the benefits of both LSM and QPSDM to improve the amplitude fidelity and image resolution of seismic data. Using an OBC data set acquired over the Angelin gas field offshore Trinidad, we demonstrate that LSQM not only retains the full benefits of QPSDM while mitigating its issue of overboosted noise but also compensates for inhomogeneous illumination caused by overburden velocity variations and irregular acquisition geometry that standard PSDM suffers from, leading to a final product with higher resolution and improved amplitude fidelity.

## Introduction

The ultimate goal of seismic migration is to retrieve the true reflectivity of the earth's subsurface structure. However, due to its inability to fully reverse the seismic wave propagation effects, standard prestack depth migration (PSDM) often suffers from issues of uneven amplitude, limited bandwidth, and migration artifacts, among others. These problems can become more obvious in the migration image as the degree of inhomogeneous subsurface illumination, residual noise in the data, and limited aperture worsens. Least-squares migration (LSM), which aims to obtain the inverse of the forward modeling operator through minimizing the square misfit between the recorded data and modeled data, was proposed to address these issues. Conventional LSM solves this least-squares problem through an iterative inversion that usually involves

many iterations of modeling (demigration) and remigration (Schuster, 1993). This is computationally very expensive and is also highly susceptible to noise in the data and velocity errors. To make LSM more efficient and applicable to real data, image-domain single-iteration LSM methods were introduced to obtain the inverse of the Hessian matrix using approaches such as point spread functions or non-stationary matching filters (Guitton, 2004; Lecomte, 2008; Valenciano et al., 2009; Fletcher et al., 2016). More recently, Wang et al. (2016) extended Guitton's matching filter method to the curvelet domain. In addition to the common benefits of LSM that compensate for illumination variations and suppress migration-related artifacts, LSM in the curvelet domain has also shown to be effective at attenuating random noise, migration swings, and other noise (Wang et al., 2016).

The amplitude fidelity and resolution of a migration image can also be negatively impacted by the anelastic absorption and elastic scattering of seismic waves as they propagate through the earth. Since high-frequency energy is attenuated more rapidly than low-frequency energy, reflectors beneath strong absorption/scattering anomalies suffer from dimmer amplitude and degraded resolution. Conventional PSDM and LSM often neglect this absorption effect, partly due to the difficulty in obtaining a reasonable Q model that can properly describe the absorption property of the subsurface. Over the last few years, Q tomography and Q migration have been developed and evolved to tackle this problem (e.g., Xin et al., 2008; Xie et al., 2009; Fletcher et al., 2012; Xin et al., 2014; Gamar et al., 2015). With a properly derived Q model, Q prestack depth migration (QPSDM) can be used to compensate for the spatially variant attenuation effects during migration, honoring the path of wave propagation (Xie et al., 2009; Fletcher et al., 2012). However, since both the Q compensation level and the noise level of seismic data usually increase with travel time and frequency, high frequency noise and migration swings are often overamplified in O migration. This boosted noise can overpower underlying weak signal, increasing the difficulty for seismic interpretation and AVO analysis.

Here we bring Q migration into the framework of LSM to complement one with the other and fully exploit the merits of both. Least-squares Q migration (LSQM) can lead to better seismic images by not only compensating for uneven illumination caused by overburden velocity variations and/or irregular acquisition geometries, but also compensating for attenuation effects while mitigating the excessive noise that typically accompanies QPSDM. We

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Figure 1: Kirchhoff migrations with 0° azimuth data: a) raw Kirchhoff stack and b) LSKir stack at one crossline section; c) raw Kirchhoff offset gathers and d) LSKir offset gathers. The maximum offset of the gathers is 6 km.

apply LSQM to a full-azimuth ocean-bottom cable (OBC) survey acquired over the Angelin gas field offshore Trinidad and demonstrate its effectiveness in improving the amplitude response, image resolution, and the signal-to-noise ratio (S/N) of reservoir reflectors beneath strong shallow absorption anomalies.

### Least-squares Q-migration

LSM inverts for the earth's reflectivity (*m*) by minimizing the difference between the recorded data ( $d_0$ ) and Bornmodeled synthetic data obtained from the reflectivity. For LSQM, a quality factor (Q) is introduced in the wave propagation to account for the anelastic absorption and elastic scattering (Wu et al., 2017):

$$\operatorname{min}_{m} \left\| d_{0} - L_{Q} m \right\|^{2}. \tag{1}$$

The standard solution to this problem can be written as:

$$m = \left(L_Q^T L_Q\right)^{-1} L_Q^T d_0, \tag{2}$$

where  $L_Q$  is the visco-acoustic Born modeling operator,  $L_Q^T$  is its adjoint operator, and  $L_Q^T L_Q$  is the so-called Hessian matrix. The computation and storage of the Hessian matrix are impractical for 3D real problems. Therefore, similar to standard LSM, Equation 2 can be solved with gradient-based iterative approaches (Schuster, 1993) or non-stationary matching filter-based single-iteration approaches (Guitton, 2004; Lecomte, 2008). In our single-iteration implementation, we approximate  $(L_Q^T L_Q)^{-1}$  with the curvelet-domain Hessian filter (CHF) approach proposed by Wang et al. (2016) to better handle angle- and frequency-dependent illumination patterns and Q effects.

We start with the initial migration image  $(m_0 = L_Q^T d_0)$  as the reflectivity model and generate a remigration image  $(m_1 = (L_Q^T L_Q)m_0)$  through acoustic Born modeling and migration. Least-squares matching filters  $(s = (L_Q^T L_Q)^{-1})$ are designed by matching  $m_1$  to  $m_0$  in the curvelet domain:

 $\min(\|\mathcal{C}(m_0) - s\mathcal{C}(m_1)\|^2 + \varepsilon \|s\|^2), \quad (3)$ 

where C is the curvelet transform operator and  $\varepsilon$  is a weighting factor for Tikhonov regularization. The output of LSQM can be written as:

$$m = C^{-1}(|s|C(L_0^T d_0)), \tag{4}$$

where  $C^{-1}$  is the inverse curvelet transform operator. We only apply the amplitude component (|s|) of the matching filter in order to preserve the kinematics of events. We note that the LSQM approach outlined here can be generally applied to different types of migrations, such as Kirchhoff and RTM. In addition,  $m_1/m$  can be either stack or surface offset gathers. In this study, CHF filters were designed and applied in the surface offset gather domain.

#### Applications to Angelin OBC data

The Angelin gas field is located in a shallow water area (water depth ~ 60 m) in the highly-faulted Columbus Basin east of Trinidad. It consists of multiple levels of gas reservoirs in different fault blocks. The shallow gas reservoirs and gas pockets can have strong absorption effects, making it very challenging to image reservoirs beneath them. In 2012, a full-azimuth OBC survey was acquired using BP's Independent Simultaneous Source (ISS®) technology (Howe et al., 2008; Abma et al., 2012). It has a dense source grid of 50 m by 50 m and a receiver spacing of 25 m along the cable and 300 m between cables. This full-azimuth OBC survey has a very high nominal fold (~ 2700) in the central area. In this study, the pre-processed up-going wavefield was used in migration. Due to the challenges in deblending, demultiple, and noise attenuation for shallow-water OBC data, a notable amount of blending noise, residual multiples, and residual noise can be observed in the pre-processed up-going data.

We first demonstrate the benefit of least-squares Kirchhoff migration (LSKir) using the  $0^{\circ}$  azimuth (along the cable direction) OBC data, where noise is more obvious due to less stacking power and stronger acquisition footprint from single azimuth input. As shown in Figures 1a and 1c, the raw Kirchhoff stack and offset gathers contained a significant amount of noise and migration swings, which severely contaminated the underlying seismic events. LSKir effectively attenuated the noise and swings and boosted the amplitude of some weak events (indicated by blue arrows) by compensating for illumination variations

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Figure 2: Kirchhoff inline stacks with the full azimuth OBC data. (a) Kirchhoff stack without Q; (b) Estimated Q model overlaid on Kirchhoff stack; (c) Stack of Q Kirchhoff migration; (d) Stack of LSQKir. (e) and (f) show the amplitude spectra of the three stacks in the left and right windows, respectively.

(Figures 1b and 1d). LSKir produced a better stack image with more balanced amplitudes and cleaner gathers. Overall, the illumination compensation effect from LSKir was not significant at the middle to great depths for the Angelin OBC data, likely because the overburden sediment does not have large velocity variations and the shot carpet is relatively dense and nearly uniform.

In the least-squares Q Kirchhoff migration (LSQKir), a 3D Q model was first derived using high-resolution volumetric Q tomography in a tilted orthorhombic velocity model (Gamar et al., 2015). In this study, the Q tomography update was limited to small depths due to the poor S/N at greater depths. Overall, the estimated Q model followed geological structures and some plausible absorption anomalies (Figure 2b). It contained strong absorption (large 1/Q value) above weak and low-frequency events and weak absorption above relatively strong and high-frequency

events. The corresponding Q Kirchhoff migration boosted event amplitude and improved image resolution by broadening the amplitude spectrum (Figure 2c). Compared to the raw PSDM stack (Figure 2a), it had more balanced amplitudes and more similar amplitude spectra between the less-attenuated and the heavily-attenuated zones (Figures 2e and 2f). However, noise and migration swings were also greatly amplified and thus smeared weak reflectors in those strongly attenuated areas, despite the high nominal fold (~2700) of the input data. In contrast, the stack image of LSQKir showed significantly reduced noise and improved reflector amplitude and continuity (Figure 2d).

The corresponding offset-vector-tile (OVT) gathers are shown in Figure 3. They further confirm Q migration boosted more higher-frequency events that often have lower S/N, resulting in a gather with a worse S/N (Figure 3b). LSQKir effectively attenuated the undesired noise and



Figure 3: Kirchhoff OVT gathers of a) PSDM, b) QPSDM, and c) LSQKir. RMS amplitude display.



Figure 4: RMS amplitude maps of a deep target horizon, extracted from Kirchhoff stacks of a) PSDM, b) QPSDM, and c) LSQKir. GWC: gas-water contact.

produced a gather with more balanced amplitudes, better resolution, and higher S/N (Figure 3c) compared to the conventional Kirchhoff PSDM gather (Figure 3a), especially for events in the blue rectangle. The RMS amplitude maps from a target horizon beneath shallow absorption bodies are presented in Figure 4. Compared to the Kirchhoff PSDM, Kirchhoff QPSDM boosted the amplitudes in the target area (highlighted by the ellipse) and produced a map with more balanced amplitudes, but the background noise became more obvious and blurred the gas-water contact (GWC) (Figure 4b). In comparison, LSQKir suppressed the excessive background noise and retained a sharp GWC (Figure 4c).

## **Conclusions and discussion**

We have demonstrated that combining LSM with QPSDM is an effective strategy to improve the amplitude fidelity and image resolution for reflectors beneath strong absorption anomalies using the Angelin OBC data. LSQM can compensate for inhomogeneous illumination associated with overburden velocity and acquisition geometry and attenuate migration artifacts/noise as standard LSM, and it can also compensate for absorption effects and mitigate the issue of excessive noise in QPSDM. The output of LSQM has improved amplitude fidelity, better resolution, and higher S/N, which can provide added value for and reduce the uncertainty of later interpretation work.

We point out that obtaining a true amplitude seismic image is a complicated matter. Uncertainties in the derived Q model will impact accurate compensation of reflector amplitudes, especially in areas with poor S/N. Furthermore, the migration amplitude is also affected by many other factors besides those considered here, such as non-focusing due to velocity errors and elastic effects that cannot be addressed by acoustic modeling. Nonetheless, LSQM is a step forward in improving the amplitude fidelity and resolution of seismic image by bringing the consideration of anelastic absorption and elastic scattering into the LSM game.

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