

Joint 4D single-iteration least-squares migration

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Summary

Time-lapse (4D) seismic monitoring generally benefits from a dedicated co-processing where important inconsistencies between vintages are attenuated. We propose a joint formulation for least-squares migration (LSM) that further mitigates effects such as low repeatability and diverse data quality, by constraining the least-squares inversion with information shared among baseline and monitor data sets. It can be applied in both post-stack and pre-stack domains. We show results for two pairs of baseline/monitor data sets with different levels of repeatability in the Brazilian pre-salt. In both cases, joint 4D LSM gave an overall reduction of 4D noise.

Introduction

Time-lapse processing is a strategic tool for reservoir management and field development. It allows for the understanding of fluid flow and changes in reservoir properties between wells, assisting decisions for extending the lifetime of producing oil fields (Lumley, 2001).

Generally consisting of vintages acquired across the span of production, factors such as survey repeatability and data quality play a significant role in designing subsequent surveys. A dedicated processing sequence is ideal for separating noise from true 4D signal and extracting the maximum value from the data.

Key steps in pre-processing are evolving to mitigate the challenges of time-lapse seismic monitoring. For instance, 4D binning (Haacke et al., 2013; Theriot et al., 2015) improves the repeatability of the surveys, reducing spurious noise coming from differences in acquisition geometry. Other pre-processing steps, such as deghosting, benefit from a joint inversion scheme, which improves the quality of the 4D signal-to-noise ratio (S/N) (Zhao et al., 2023).

On the imaging side, many of the advances relate to obtaining separate velocity models through an appropriate strategy of 4D full-waveform inversion (FWI) (see Fu and Innanen, 2023, for a review). A recent development poses FWI as a joint problem at each iteration with shared information among the vintages (Rodrigues et al., 2024). Promising results were obtained in the Brazilian pre-salt with a hybrid OBN over NATS setup. These efforts highlight the increasing trend in using FWI directly for imaging and interpretation, complementing the traditional imaging from LSM (Izeli et al., 2024).

Nonetheless, LSM remains an important tool for time-lapse monitoring. Unlike LSM, FWI is a non-linear process that requires many iterations to converge. Therefore, 4D FWI is computationally much more costly than 4D LSM. When high frequencies are needed for reservoir characterization, the cost of 4D FWI might quickly become prohibitive. Moreover, LSM methods can be extended to the pre-stack domain without much additional effort.

Building on previous work in the context of 4D FWI (Rodrigues et al., 2024), we formulate LSM for time-lapse data as a joint inversion problem by sharing information among the vintages. Results are shown for a couple of pairs of baseline/monitor data sets in the Brazilian pre-salt. The first is a hybrid NATS-OBN pair. In this case, uplifts from the proposed approach are visible already in the stack, given the low repeatability between acquisitions. In the second example, consisting of an OBN-OBN pair, uplifts are more subtle, given the high repeatability of this type of acquisition, and are more noticeable in the pre-stack domain.

Joint 4D LSM

LSM inverts for the reflectivity m that fits the observed data d through linearized Born modeling L , or demigration. The problem is posed as an inversion with the least-squares misfit

$$f(m) = \frac{1}{2} \|d - Lm\|^2. \quad (1)$$

The solution involves the computation of the inverse of the Hessian matrix $H = L^T L$, where L^T is the migration operator:

$$m = (L^T L)^{-1} L^T d. \quad (2)$$

For real 3D problems, the Hessian is very large and direct computation of its inverse is not possible. Different strategies have been tried to approximate the inverse: both iterative (Schuster, 1993; Nemeth et al., 1999; Tang, 2008) and single-iteration (Hu et al., 2001; Rickett, 2003; Guitton, 2004; Lecomte, 2008) schemes. In this work, we follow the single-iteration approach of Wang et al. (2016) based on curvelet domain filters.

Let $m_1 = L^T L m_0$ be the Hessian product (demigration-remigration) of a reference reflectivity model m_0 . The inverse Hessian is computed through a proxy filter, s , in the curvelet domain by minimizing the cost function

$$f(s) = \|C(m_0) - sC(m_1)\|^2 + \epsilon \|s\|^2, \quad (3)$$

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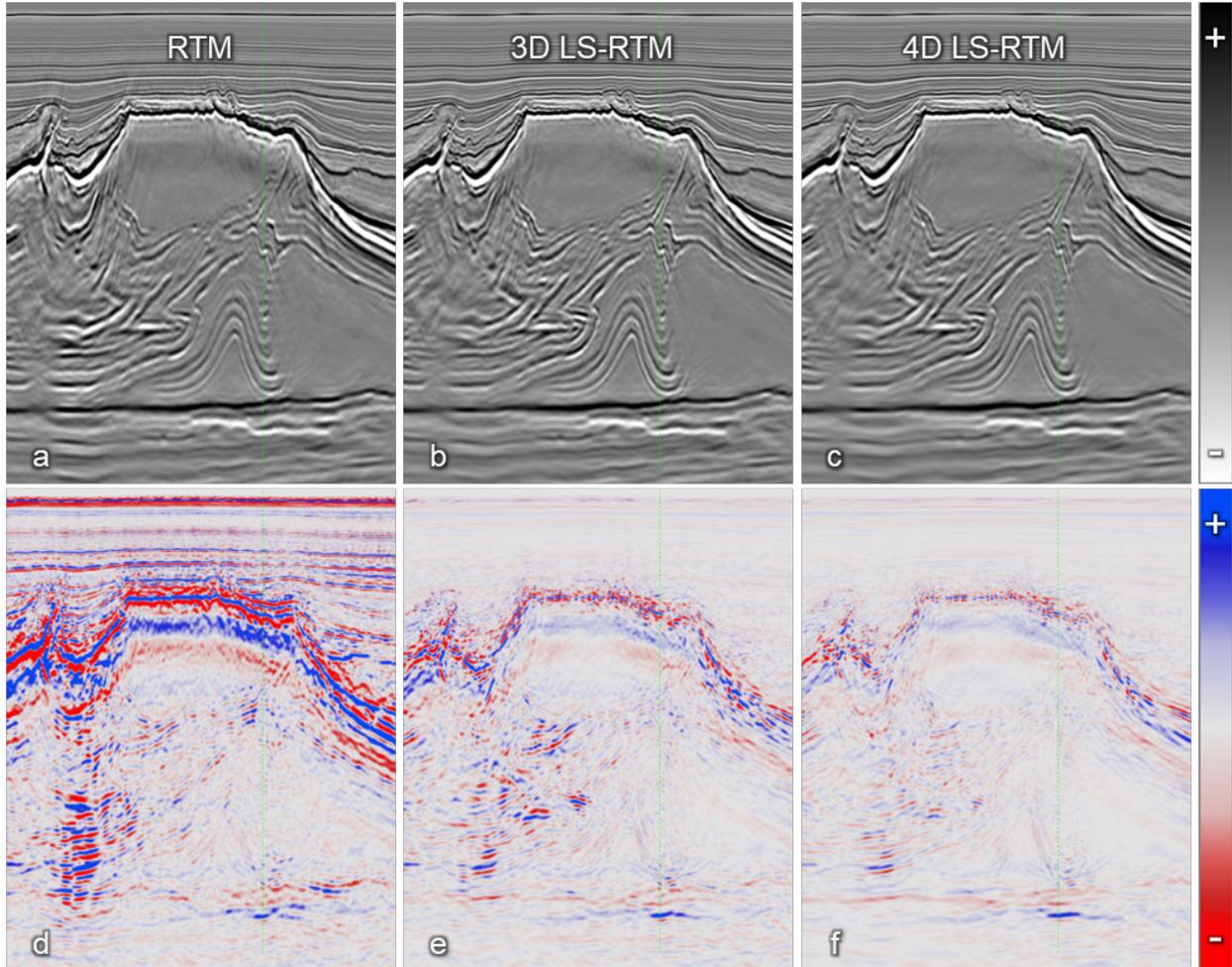


Figure 1: Application on NATS-OBN hybrid pair. (a) RTM image of NATS baseline, (b) independent and (c) joint baseline LS-RTM. 4D difference between monitor and baseline (d) RTM, (e) independent and (f) joint LS-RTM.

where C is the curvelet transform operator and ϵ is a hyperparameter controlling the regularization of the filter. The solution for the LSM is

$$m = C^{-1}(|s|C(L^T d)), \quad (4)$$

where C^{-1} is the inverse curvelet transform and only the amplitude of the filter is applied to avoid introducing any phase effects. Wang et al. (2017a) applied the same framework in the context of 4D LSM, using two separate cost functions for inverting the two Hessians $H_B = L_B^T L_B$ and $H_M = L_M^T L_M$ of baseline and monitor, but using a common reference reflectivity m_0 .

As in Rodrigues et al. (2024), we expand on this idea by introducing a single cost function with a coupling term,

which constrains the solutions of both vintages to a common ground:

$$f(s_B, s_M) = \|C(m_0) - s_B C(m_{1B})\|^2 + \epsilon_B \|s_B\|^2 + \|C(m_0) - s_M C(m_{1M})\|^2 + \epsilon_M \|s_M\|^2 + (\langle \lambda, s_B C(m_{1B}) - s_M C(m_{1M}) \rangle + c.c.), \quad (5)$$

where $m_{1B} = L_B^T L_B m_0$ and $m_{1M} = L_M^T L_M m_0$ are the baseline and monitor Hessian products on m_0 . In the parenthesis, *c.c.* denotes the complex conjugate of the preceding expression and λ is a Lagrange multiplier imposing the constraint

$$s_B C(m_{1B}) = s_M C(m_{1M}). \quad (6)$$

This condition makes the curvelet Hessian filters share

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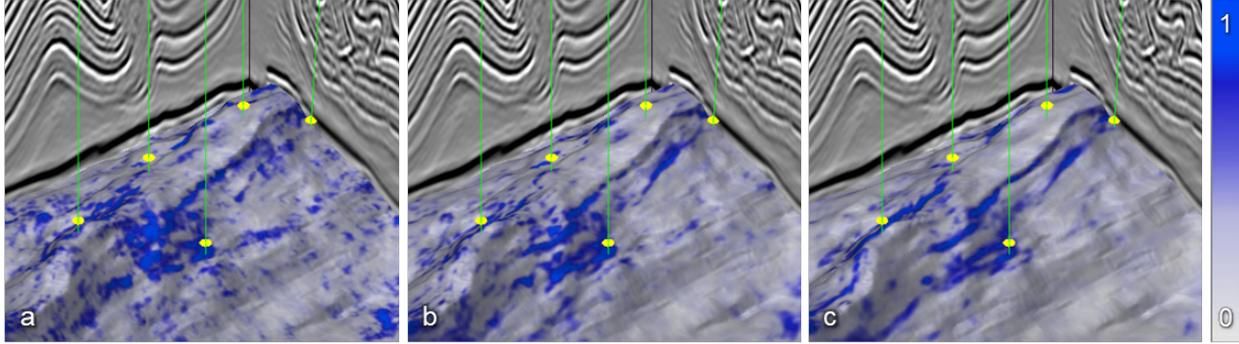


Figure 2: RMS of the 4D signal extracted at the reservoir level relative to the baseline RMS for (a) RTM, (b) independent and (c) joint LS-RTM.

information of the Hessians of both vintages. Since the Hessian captures information of the geometry of acquisition (Pratt et al., 1998), we are imposing the limitations of one survey on the other. This implies, for instance, that, in curvelet domain, the null-space of the monitor Hessian H_M , which is captured by $C(m_{1M})$, will be inherited by the baseline filter s_B . The opposite is also true, making both solutions share the same extended null-space. The results

$$\begin{aligned} m_B &= C^{-1}\left(|s_B|C(L_B^T d_B)\right), \\ m_M &= C^{-1}\left(|s_M|C(L_M^T d_M)\right), \end{aligned} \quad (7)$$

are the separate application of the filters in the respective raw migrations $L_B^T d_B$ and $L_M^T d_M$ of baseline and monitor. Note that the method can be naturally extended to be used as a preconditioning for iterative LSM (Wang et al., 2017b).

The method can also be extended to the pre-stack domain (Wang et al., 2016). The formulation is the same: for each instance i of the pre-stack domain (SOG, subsurface angle, etc.), we seek filters s_B^i and s_M^i which minimize the misfit

$$\begin{aligned} f(s_B^i, s_M^i) &= \|C(m_0) - s_B C(m_{1B}^i)\|^2 + \epsilon_B \|s_B^i\|^2 \\ &+ \|C(m_0) - s_M C(m_{1M}^i)\|^2 + \epsilon_M \|s_M^i\|^2 \\ &+ (\lambda_i, s_B^i C(m_{1B}^i) - s_M C(m_{1M}^i)) + c.c.), \end{aligned} \quad (8)$$

where $m_{1B}^i = L_B^{iT} L_B^i m_0$ and $m_{1M}^i = L_M^{iT} L_M^i m_0$ are the pre-stack demigration-remigration for baseline and monitor of a common reference m_0 used for all instances i . Similarly, the LSM solutions are

$$\begin{aligned} m_B^i &= C^{-1}\left(|s_B^i|C(L_B^{iT} d_B^i)\right), \\ m_M^i &= C^{-1}\left(|s_M^i|C(L_M^{iT} d_M^i)\right), \end{aligned} \quad (9)$$

the filters applied to the pre-stack migrations $L_B^{iT} d_B^i$ and $L_M^{iT} d_M^i$.

Results

We apply the proposed method to data from a Brazilian pre-salt field, with a NATS baseline survey acquired before start of production, and two subsequent monitoring OBN (OBN₁ and OBN₂) data sets acquired during production. The geological context of the area can be identified in the reverse time migration (RTM) image in Figure 1a. Deep carbonate reservoirs are located below thick and stratified salt. This type of reservoir is challenging for time-lapse imaging due to the typical stiffness of the carbonates in the region, which may lead to relatively subtle 4D differences (Cypriano et al., 2019; Cruz et al., 2021).

We first study the hybrid pair (NATS-OBN₁). The data sets went through 4D pre-processing. In particular, the OBN was limited to the NATS acquisition azimuth to attenuate the lack of repeatability. One might expect that LSM would be enough to solve this type of geometric non-repeatability, but it is an ill-posed problem with many possible solutions. Increasing the similarity between acquisitions helps ensure more compatible solutions for the vintages. The top row of Figure 1 displays a section of the RTM, the independent (3D) LS-RTM and the joint (4D) LS-RTM images of the NATS data. The effect of the joint inversion on the baseline image is not evident when comparing Figures 1b and 1c; it becomes more clear looking at the 4D differences in the bottom row. Comparing Figures 1d and 1e, independent LS-RTM balances the amplitudes of the images, rendering a cleaner 4D difference than RTM. The result of joint LS-RTM difference in Figure 1f provides further reduction of 4D noise, because of the shared geometrical limitations in the calculation of the Hessian filters. We observe noise reduction in the overburden, particularly so in the intra-salt, and also at the reservoir level, better delineating possible 4D signals.

Figure 2 shows a 3D section with a horizon at the reservoir level, together with a few well paths. We plotted the normalized RMS of the 4D difference relative to the baseline

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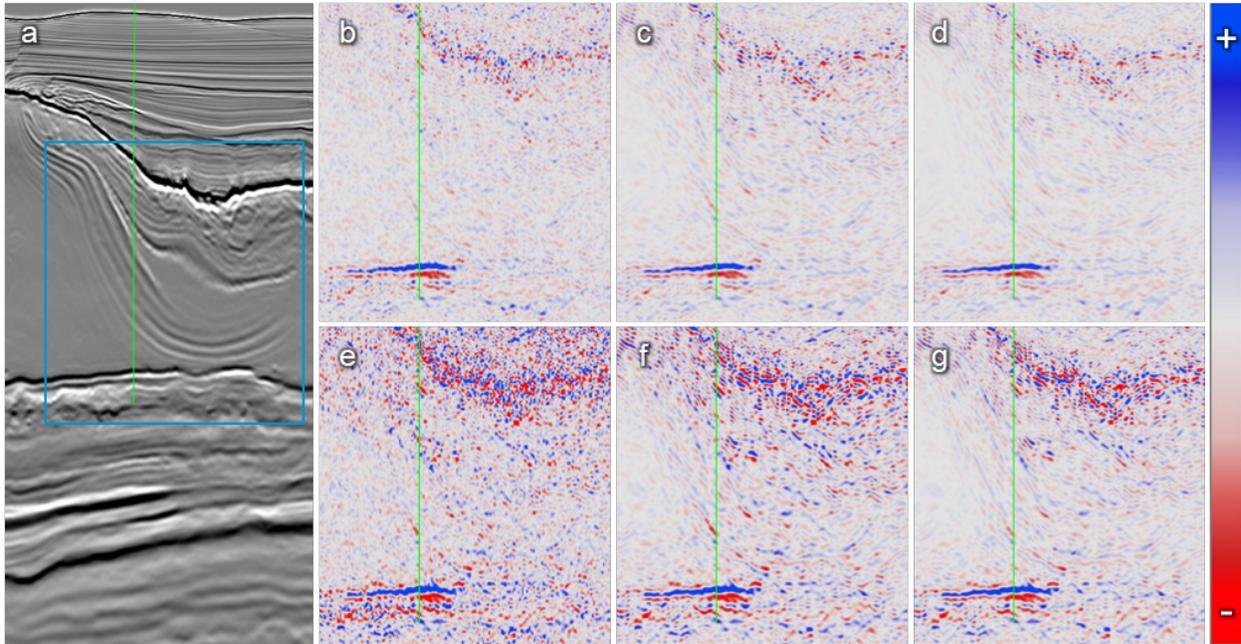


Figure 3: Pre-stack application on OBN-OBN pair. (a) Full stack of the RTM image of the baseline. 4D differences for full stack (b) RTM, (c) independent LS-RTM and (d) joint LS-RTM. 4D differences for near stack (e) RTM, (f) independent LS-RTM and (g) joint LS-RTM.

RMS. Comparing RTM (Figure 2a) to independent LS-RTM (Figure 2b) and then joint LS-RTM (Figure 2c), an incremental decrease in noise level can be noted, while the signals around the wells remain preserved. Moreover, a better continuity of the signal is observed with the joint formulation. Globally, the median NRMS based on RTM is about 18% at the reservoir level, and independent LS-RTM reduces it to 15%. With the joint LS-RTM, the NRMS drops to 12%.

Next, we apply the method to the second pair of vintages (OBN₁-OBN₂). The data sets went through 4D pre-processing as well. Although geometric repeatability is not an issue, water column variations needed to be properly corrected. In Figure 3a, we show a section of the RTM image of the OBN₁ with a well path for reference. Figures 3b, 3c, and 3d show a zoom-in of the differences of the full-stack for the RTM, independent LS-RTM, and joint LS-RTM, respectively, and Figures 3e, 3f, and 3g show the same displays for a near-stack.

On the top row of Figure 3, we notice a clear improvement of the S/N from RTM to independent LS-RTM at the reservoir level, and a subtle improvement from independent to joint LS-RTM. A greater impact from LS-RTM, both independent and joint, can be noticed in the bottom row for a near stack. The comparison of Figures 3f and 3g shows overall reduced noise levels for the joint LS-RTM, more

clearly at the top of salt and intra-salt, while preserving the possible 4D signal at the base of salt. Despite the visual perception, in both cases the NRMS reduction is similar. In the full stack, RTM shows a median NRMS of 3.1% at the reservoir level. Independent LS-RTM reduces it to 2.6% and the joint formulation to 2.4%. The figures for the near stack follow the same pattern: 4.5% for RTM, 3.7% for independent LS-RTM, and 3.4% for joint LS-RTM.

Conclusions

We have proposed a joint formulation for 4D single-iteration LSM by imposing some additional constraints in the calculation of the inverse Hessian filters. These constraints enforce that the geometrical information contained in the Hessian matrix is shared among different vintages, thus providing a more consistent 4D image.

The method was applied to a NATS-OBN hybrid pair of data sets from the Brazilian pre-salt as well as to an OBN-OBN pair in the same area. Regardless of the level of acquisition repeatability, an overall reduction in 4D noise was observed.

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